

## Leptonic CP violation: zero, maximal or between the two extremes

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**ABSTRACT:** Discovery of the CP-violation in the lepton sector is one of the challenges of the particle physics. We search for possible principles, symmetries and phenomenological relations that can lead to particular values of the CP-violating Dirac phase,  $\delta$ . In this connection we discuss two extreme cases: the zero phase,  $\delta = 0$ , and the maximal CP-violation,  $\delta = \pm\pi/2$ , and relate them to the peculiar pattern of the neutrino mixing. The maximal CP-violation can be related to the  $\nu_\mu - \nu_\tau$  reflection symmetry. We study various aspects of this symmetry and introduce a generalized reflection symmetry that can lead to an arbitrary phase that depends on the parameter of the symmetry transformation. The generalized reflection symmetry predicts a simple relation between the Dirac and Majorana phases. We also consider the possibility of certain relations between the CP-violating phases in the quark and lepton sectors.

**KEYWORDS:** CP violation, Neutrino Physics.

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## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. Rephasing invariants and criteria for conservation of CP</b>	<b>2</b>
<b>3. Zero Dirac CP-violating phase</b>	<b>5</b>
3.1 Zero off-diagonal elements of $h_{\alpha\beta}$	6
3.2 Small Dirac phase	10
3.3 Zero Dirac phase with nonzero elements of $h_{\alpha\beta}$	11
<b>4. Maximal CP violating phase</b>	<b>13</b>
4.1 $\mu - \tau$ reflection symmetry	13
4.2 $\mu - \tau$ exchange versus $\mu - \tau$ reflection symmetry	16
4.3 Renormalization group effects	17
4.4 $A_4$ symmetry and maximal CP violation	18
<b>5. Nonzero CP violating phase</b>	<b>18</b>
5.1 Generalized $\mu - \tau$ reflection symmetry	19
5.2 Seesaw mechanism and leptogenesis	20
<b>6. Possible relations between the phases of the CKM and PMNS matrices</b>	<b>21</b>
<b>7. Conclusion</b>	<b>22</b>
<b>A. Neutrino mass square matrix</b>	<b>24</b>

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## 1. Introduction

Observation of the effects of the CP violation was one of the fundamental discoveries in physics [1] in the past century. The violation of the CP-symmetry is established in the quark sector and it is natural to expect that CP violation occurs in the lepton sector, too. Furthermore, in the lepton sector one may even find additional sources of CP-violation; e.g., the Majorana phases of neutrino masses, the right handed (RH) neutrino mass matrix in the context of seesaw mechanism, or mixing with new neutrino states.

What can we learn from measurements of the CP-violating phases? What is the underlying physics? These questions are imperative especially in view of development of the challenging and rather expensive experimental programs to measure the CP-violation in the neutrino oscillations [2]. What would the possible implications of the future measurements of the phase be?

We still have no theory of CP-violation in the quark sector that would explain the observed value of the phase  $\delta_{\text{CKM}}$ . What can we say about the CP-violation in the lepton sector where the information about masses and mixings is not so complete as in the quark sector? Can the situation be simpler here?

In view of this incomplete knowledge, we can take the following routes to approach the questions raised above.

1. Some extreme situations can be studied; e.g., the possibility of zero Dirac phase,  $\delta = 0$ , or maximal phase,  $\delta = \pi/2$ .
2. We can also try to relate the CP-violating phases in the quark and lepton sectors. In this line, one may ask if the phases can be equal or complementary; or if there is another simple relation between the phases.

There are few specific models of neutrino mass and mixing that predict the value of the CP-violating phase. For example, the Zee model (whose minimal version is excluded by the recent data) predicts a neutrino mass matrix invariant under CP. In the models with  $A_4$  symmetry the maximal value for the Dirac phase,  $\delta = \pi/2$ , appears [3]. Some structures of the mass matrices lead to maximal mixing [4]. Classification of matrices with a certain number of texture zeros in the flavor basis and the corresponding CP-violation effects have been considered in [5]. However, till now no systematic study of the CP-violation exists. In this paper, we search for principles, symmetries as well as phenomenological and empirical relations that lead to particular values of the CP-violating phase. In section 2, we study some general properties of the neutrino mass matrix and formulate criteria for the CP-violation. In section 3, we search for symmetries which predict  $\delta = 0$  and discuss the related phenomenological consequences. In section 4, we consider the case of maximal CP-violation. In section 5, we formulate conditions that lead to certain values of the phase which differ from zero and  $\pi/2$ . The possibility of relations between the quark and lepton CP-violating phases is studied in section 6. In section 7, we summarize our results.

## 2. Rephasing invariants and criteria for conservation of CP

We assume that there are only three light Majorana neutrinos and parameterize the neutrino mass matrix in the flavor basis  $\nu_f \equiv (\nu_e, \nu_\mu, \nu_\tau)$  as

$$m_\nu = \begin{bmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ \dots & m_{\mu\mu} & m_{\mu\tau} \\ \dots & \dots & m_{\tau\tau} \end{bmatrix}. \quad (2.1)$$

We will use the standard parametrization of the mixing matrix which diagonalizes  $m_\nu$

$$U_{PMNS} = U_{23}(\theta_{23})\Gamma_\delta U_{13}(\theta_{13})\Gamma_{-\delta} U_{12}(\theta_{12})\Gamma_M. \quad (2.2)$$

Here  $U_{ij}(\theta_{ij})$  is the matrix of rotation by an angle  $\theta_{ij}$  in the  $ij$ -plane;

$$\Gamma_\delta \equiv \text{diag}(1, 1, e^{i\delta}), \quad \Gamma_M \equiv \text{diag}(1, e^{\phi_2/2}, e^{\phi_3/2}), \quad (2.3)$$

where  $\phi_2, \phi_3$  are the Majorana phases defined in such a way that mass eigenvalues are made real and positive. These phases could be included in the definition of the eigenvalues  $m_i$ , and since the oscillation probabilities are determined by  $|m_i|^2$ , only the phase  $\delta$  affects oscillations.

Let us introduce the following matrix

$$h \equiv m_\nu \cdot m_\nu^\dagger. \quad (2.4)$$

The CP-violation in neutrino oscillations is then determined by [6]

$$\mathcal{J} = \text{Im}[h_{e\mu}h_{\mu\tau}h_{\tau e}] \quad (2.5)$$

which is related to the Jarlskog invariant,  $J_{CP}$ , as

$$\mathcal{J} = \Delta m_{12}^2 \Delta m_{32}^2 \Delta m_{13}^2 J_{CP}. \quad (2.6)$$

Here

$$J_{CP} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin \delta \quad (2.7)$$

and  $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ ,  $s_{12} \equiv \sin \theta_{12}$ ,  $c_{12} \equiv \cos \theta_{12}$ , *etc.* In terms of  $m_\nu$  the elements  $h_{\alpha\beta}$  are given by

$$\begin{aligned} h_{e\mu} &= m_{ee}m_{e\mu}^* + m_{e\mu}m_{\mu\mu}^* + m_{e\tau}m_{\tau\mu}^* \\ h_{\mu\tau} &= m_{\mu e}m_{e\tau}^* + m_{\mu\mu}m_{\mu\tau}^* + m_{\mu\tau}m_{\tau\tau}^* \\ h_{\tau e} &= m_{\tau e}m_{ee}^* + m_{\tau\mu}m_{\mu e}^* + m_{\tau\tau}m_{\tau e}^*. \end{aligned} \quad (2.8)$$

Explicit expressions for  $h_{\alpha\beta}$  in terms of oscillation parameters are given in the appendix. Notice that  $h$  does not depend on the Majorana phases and therefore provides a test of the Dirac phase without any ambiguity from the Majorana phases.

In order to investigate symmetries and relations that determine the CP-violation in general (either through the Majorana phases or through the Dirac phase) we should consider the matrix  $m_\nu$  rather than  $h$ . However, the arguments of the elements of  $m_\nu$  change with rephasing the fields and therefore are not physical. To perform such an analysis, we should use rephasing invariant quantities [7]. Under rephasing of neutrino states,  $(\nu_e, \nu_\mu, \nu_\tau) \rightarrow (e^{i\alpha_e}\nu_e, e^{i\alpha_\mu}\nu_\mu, e^{i\alpha_\tau}\nu_\tau)$ , the matrix changes as

$$m_\nu \rightarrow \text{diag}[e^{i\alpha_e}, e^{i\alpha_\mu}, e^{i\alpha_\tau}] \cdot m_\nu \cdot \text{diag}[e^{i\alpha_e}, e^{i\alpha_\mu}, e^{i\alpha_\tau}]. \quad (2.9)$$

Apparently, the following combinations of matrix elements are rephasing invariant:

$$I_1 \equiv m_{e\mu}^2 m_{ee}^* m_{\mu\mu}^*, \quad I_2 \equiv m_{e\tau}^2 m_{ee}^* m_{\tau\tau}^*, \quad I_3 \equiv m_{\mu\tau}^2 m_{\mu\mu}^* m_{\tau\tau}^*, \quad (2.10)$$

so are the equivalent combinations:  $m_{\alpha\beta}^2/m_{\beta\beta}m_{\alpha\alpha}$ , where  $\alpha \neq \beta$ . Notice that under the  $\mu - \tau$  permutation,  $I_3$  is invariant and

$$I_1 \leftrightarrow I_2. \quad (2.11)$$

Two other invariants read as

$$I_4 \equiv \frac{m_{e\mu}^2 m_{\mu\mu}^*}{m_{e\tau}^2 m_{\tau\tau}^*} = \frac{I_1}{I_2} \quad (2.12)$$

or  $(m_{e\mu} m_{e\tau}^*)^2 m_{\tau\tau} m_{\mu\mu}^*$  and

$$I_5 \equiv \frac{m_{e\tau} m_{\mu\tau} m_{\tau\tau}^*}{m_{e\mu}} = \sqrt{\frac{I_3 I_2}{I_1}}. \quad (2.13)$$

Let us first formulate conditions for the complete CP-conservation; *i.e.*, the case that the Dirac and Majorana phases are both zero. For chiral states the CP transformation coincides with C-conjugation:

$$\nu_\alpha \rightarrow \nu_\alpha^C \equiv C \bar{\nu}_\alpha^T, \quad (2.14)$$

where  $C \equiv i\gamma_2\gamma_0$ . In terms of the charge conjugated states, the mass terms (together with hermitian conjugate) can be written as

$$m_{\alpha\beta} \nu_\alpha^T C \nu_\beta + m_{\alpha\beta}^* \nu_\alpha^C C \nu_\beta^*. \quad (2.15)$$

Under CP transformations  $\nu_\alpha \rightarrow \nu_\alpha^c$  (an additional phase can be removed by rephasing), and therefore  $m_{\alpha\beta} \rightarrow m_{\alpha\beta}^*$ . Consequently, the neutrino mass terms are CP invariant if

$$m_{\alpha\beta} = m_{\alpha\beta}^*, \quad (2.16)$$

that is, if the matrix elements can be made real after rephasing.

Using invariants  $I_i$  we can formulate the CP invariance in the rephasing independent form. Similar analysis has been performed recently in [8], although, as we will see, some differences between the two exist.<sup>1</sup> In particular, while [8] focuses on the textures compatible with neutrino data our approach is more general and can find application in contexts beyond the neutrino physics. Moreover, the aim of [8] is to formulate measures for CP-violation rather than formulating criteria for CP-violation.

1. In the case that all the diagonal entries of the mass matrix are nonzero, CP is conserved if and only if the three invariants  $I_1, I_2, I_5$  are real:

$$\text{Im}I_1 = 0, \quad \text{Im}I_2 = 0, \quad \text{Im}I_5 = 0. \quad (2.17)$$

Notice that although  $I_5$  can be written in terms of  $I_1, I_2$  and  $I_3$ , one cannot replace the condition of  $\text{Im}I_5 = 0$  with  $\text{Im}I_3 = 0$ . This is illustrated by the following matrix

$$m_\nu = \begin{bmatrix} a & e & f \\ \dots & b & id \\ \dots & \dots & c \end{bmatrix} \quad (2.18)$$

with real  $a, e, f, b, c$  and  $d$ . This matrix explicitly violates the CP but  $I_1, I_2$  and  $I_3$  associated with it are all real.

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<sup>1</sup>Our results have been presented in [9].

2. In the case that  $m_{ee} = 0$  but  $m_{\mu\mu}$  and  $m_{\tau\tau}$  are nonzero, CP is conserved if and only if the two invariants  $I_3$  and  $I_5$  are real:

$$\text{Im}I_3 = 0, \quad \text{Im}I_5 = 0. \quad (2.19)$$

3. In the case that  $m_{ee} = m_{\mu\mu} = 0$  but  $m_{\tau\tau}$  is nonzero, CP is conserved if and only if  $I_5$  is real

$$\text{Im}I_5 = 0. \quad (2.20)$$

4. Finally, if  $m_{ee} = m_{\mu\mu} = m_{\tau\tau} = 0$ , CP is conserved.

It is easy to check that conditions (2.17), (2.19), (2.20) guarantee that the mass matrix can be made real by rephasing and therefore the CP symmetry is completely conserved.

In order to test the violation of CP, one may choose to examine another set of three independent combinations of  $I_1$ ,  $I_2$  and  $I_3$ . So in this sense our criterion is not unique: depending on the type of the texture, calculating a certain set of invariants may have advantages over other sets. However, one must be aware of the possibilities such as (2.18).

Combining  $I_1$ ,  $I_2$  and  $I_3$ , one can write new forms of rephasing invariants:

$$\begin{aligned} J_1 &= m_{e\mu}m_{e\tau}m_{ee}^*m_{\mu\tau}^* = \sqrt{I_1I_2I_3^*} / (|m_{\mu\mu}|^2|m_{\tau\tau}|^2) ; \\ J_2 &= m_{e\mu}m_{\mu\tau}m_{e\tau}^*m_{\mu\mu}^* = \sqrt{I_1I_3I_2^*} / (|m_{ee}|^2|m_{\tau\tau}|^2) ; \\ J_3 &= m_{e\tau}m_{\mu\tau}m_{e\mu}^*m_{\tau\tau}^* = \sqrt{I_2I_3I_1^*} / (|m_{ee}|^2|m_{\mu\mu}|^2) . \end{aligned}$$

In fact, using (2.5) it is straightforward to show that  $\mathcal{J}$  can be written as the imaginary part of a combination of the above invariants with real coefficient such as  $|m_{\alpha\beta}|^2$ . As a result, the realness of the above rephasing invariants guarantees that the Jarlskog invariant vanishes. However, the opposite is not correct; *i.e.*, we can have  $\mathcal{J} = 0$  but some of  $I_i$  and  $J_i$  may be complex. This is due to the fact that, in contrast to the Jarlskog invariant, the  $I_i$  and  $J_i$  contain information not only on the Dirac phase but also on the Majorana phases.

### 3. Zero Dirac CP-violating phase

Due to the fact that according to the neutrino oscillation data, all the mass splitting,  $\Delta m_{ij}^2$  ( $i \neq j$ ), as well as the values of  $\sin 2\theta_{12}$ ,  $\sin 2\theta_{23}$  and  $\cos \theta_{13}$  are nonzero, the equality  $\mathcal{J} = 0$  implies  $\sin \theta_{13} \sin \delta = 0$  [see eqs. (2.6), (2.7)]. Below we derive the condition on the mass matrix that yield zero  $\sin \delta$  (*i.e.*,  $\delta = k\pi$  with integer  $k$ ) while keeping  $\sin \theta_{13}$  and Majorana phases nonzero. The possibility under consideration corresponds to a situation that the forthcoming reactor and accelerator experiments measure  $\theta_{13}$ ; however, the subsequent CP-violation searches would report a null result. What can we learn from a sizeable  $\sin \theta_{13}$  but very small CP-violation,  $J_{CP} \ll s_{13}s_{12}c_{12}s_{23}c_{23}$ ?

Before deriving the necessary condition for  $\delta = k\pi$ , let us formulate a criterion for zero  $\sin \theta_{13}$ . It is straightforward to show that if  $\sin \theta_{13} = 0$ ,

$$\tan 2\theta_{23} = \frac{2|h_{\mu\tau}|}{h_{\tau\tau} - h_{\mu\mu}}$$

and

$$\tan \theta_{23} = \frac{|h_{e\tau}|}{|h_{e\mu}|},$$

where  $h$  is defined in eq. (2.4). So the equality

$$\frac{2|h_{\mu\tau}|}{h_{\tau\tau} - h_{\mu\mu}} = \frac{2|h_{e\tau}h_{e\mu}|}{|h_{e\mu}|^2 - |h_{e\tau}|^2} \quad (3.1)$$

can be considered as a test for  $\sin \theta_{13} = 0$ . This criterion will be useful to check whether zero  $\mathcal{J}$  implies  $\sin \theta_{13} = 0$  or  $\sin \delta = 0$ .

Below we consider different situations that yield vanishing  $\sin \delta$ .

### 3.1 Zero off-diagonal elements of $h_{\alpha\beta}$

A trivial way to satisfy the condition  $\mathcal{J} = 0$  is to have one or more vanishing off-diagonal elements of  $h$ :

$$h_{\alpha\beta} = 0, \quad (\alpha \neq \beta). \quad (3.2)$$

This condition is both rephasing and parametrization invariant. In terms of the elements of the mass matrix, it implies ‘‘orthogonality’’ of the  $\alpha$  and  $\beta$  lines of the mass matrix:

$$\sum_i m_{\alpha i} m_{\beta i}^* = 0. \quad (3.3)$$

Apparently, these conditions are not necessary for  $\mathcal{J} = 0$  and therefore lead to certain predictions for  $\sin \theta_{13}$ . The hope is that a symmetry or principle will be uncovered that leads to the equality (3.2).

Let us consider three possibilities for different  $\alpha, \beta$  one by one.

1.  $h_{e\mu} = 0$ . Using expression (A.3) from the appendix and taking into account the known experimental information that 1-2 and 2-3 mixings as well as  $\Delta m_{ij}^2$  are nonzero we find that  $|h_{e\mu}| = 0$ , if

$$\delta = 0 \quad \text{and} \quad s_{13}s_{23}\Delta m_{31}^2 + \Delta m_{21}^2(s_{12}c_{12}c_{23} - s_{13}s_{23}s_{12}^2) = 0, \quad (3.4)$$

which implies a rather small value for the 1-3 mixing:

$$\sin \theta_{13} \approx -0.5 \sin 2\theta_{12} \cot \theta_{23} r_{\Delta}, \quad (3.5)$$

where

$$r_{\Delta} \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}. \quad (3.6)$$

Numerically we obtain  $|\sin \theta_{13}| \sim 0.016$ . Unfortunately, such small values of  $s_{13}$  are beyond the reach of upcoming reactor experiments such as Double CHOOZ [10] and Daya Bay [11] as well as long baseline experiments [12] (see, however, [13]). Thus, a positive result in these experiments will exclude such a possibility. Another solution is  $\delta = \pi$  and  $\sin \theta_{13} \approx 0.5 \sin 2\theta_{12} \cot \theta_{23} r_{\Delta}$ .

So, if there is a symmetry or principle that leads to  $h_{e\mu} = 0$ , this equality together with phenomenological input (nonzero masses and two mixings) imply  $\delta = 0$  and a

value for  $\sin\theta_{13}$  given in eq. (3.5). Inversely, confirming relation (3.5) will testify for such an underlying symmetry. However it will not be sufficient to conclude that  $\sin\delta = 0$ . For  $\sin\theta_{13}$  given in (3.5),  $h_{e\mu}$  vanishes only if  $\sin\delta$  also vanishes.

In terms of the elements of the mass matrix, the condition  $h_{e\mu} = 0$  can be written as

$$m_{e\tau} = -\frac{1}{m_{\mu\tau}^*}(m_{ee}m_{e\mu}^* + m_{e\mu}m_{\mu\mu}^*). \quad (3.7)$$

Although eq. (3.7) involves all the elements of the mass matrix except  $m_{\tau\tau}$  it does not, in general, imply a certain symmetry. However, as we show below, in some particular cases, simple patterns of the mass matrix emerge.

For the normal hierarchy case, eq. (3.7) yields the following form for the neutrino mass matrix

$$m_\nu = \begin{bmatrix} g & k & -k - [gk^*/A^*(1 + \epsilon_2^*)] + k\epsilon_2^* \\ \dots & A & A(1 + \epsilon_2) \\ \dots & \dots & A(1 + \epsilon_3) \end{bmatrix}, \quad (3.8)$$

where  $k, g \ll A$  and  $\epsilon_i \ll 1$ . Let us consider the special case of  $g = 0$  and  $\epsilon_2 = 0$ , so that the matrix reduces to

$$m_\nu \simeq \begin{bmatrix} 0 & k & -k \\ \dots & A & A \\ \dots & \dots & A(1 + \epsilon) \end{bmatrix}. \quad (3.9)$$

Apparently, the above matrix has an approximate  $\nu_\mu - \nu_\tau$  symmetry broken in the 2-3 block. The parameters  $k$  and  $A$  can be made real by rephasing.

The matrix (3.9) can be well motivated by a certain symmetry. If  $\epsilon = 0$ , the matrix is invariant under the following transformations

$$\begin{bmatrix} \nu_\mu \\ \nu_\tau \end{bmatrix} \Rightarrow e^{-i\theta} \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_\mu \\ \nu_\tau \end{bmatrix}, \quad (3.10)$$

$$\nu_e \Rightarrow e^{2i\theta}\nu_e \quad (3.11)$$

with arbitrary  $\theta$ . Invariance under (3.11) leads to a vanishing  $ee$ - element. The special case of  $\theta = \pi/2$  corresponds to the  $\nu_\mu \leftrightarrow \nu_\tau$  and  $\nu_e \rightarrow -\nu_e$  symmetry which implies  $m_{e\mu} = -m_{e\tau}$  and  $m_{\mu\mu} = m_{\tau\tau}$ . Finally, the invariance under general transformations (3.10) leads to  $m_{\mu\mu} = m_{\mu\tau}$ . Notice that the equality of  $m_{\mu\mu}$  and  $m_{\mu\tau}$  does not follow from the  $\mu - \tau$  permutation symmetry.

The symmetry under (3.10), (3.11) does not explain the hierarchy  $k/A \sim 0.1$ . This can be either accidental (notice that the hierarchy is not too large) or a consequence of approximate  $L_e$  conservation. Nonzero  $\epsilon$  (which is necessary for explaining the neutrino data) breaks the symmetry under (3.10). Notice that in the charged lepton sector (as well as in the quark sector) the symmetry is broken by a similar structure; *i.e.*,  $m_\tau \gg m_\mu$ . The two can be related. In this connection one can consider two different contributions to the neutrino mass matrix such that while the dominant



contribution obeys the symmetry under (3.10), (3.11), the subdominant contribution, which violates the symmetry, has a hierarchical structure similar to the one in the charged lepton sector.

Let us consider the phenomenological consequences of matrix (3.9). Diagonalization leads to the mixing angles

$$\tan 2\theta_{12} = \frac{4\sqrt{2}k}{|\epsilon|A}, \quad \sin \theta_{13} = -\frac{k|\epsilon|}{4\sqrt{2}A} = -\frac{|\epsilon|^2}{32} \tan 2\theta_{12}, \quad (3.12)$$

and  $\delta = 0$ , which is expected because  $\theta_{13}$  is nonzero. Moreover,

$$\theta_{23} = \frac{\pi}{4} - \frac{\text{Re}[\epsilon]}{4} - \frac{|\epsilon|^2}{8}.$$

For the mass eigenvalues we obtain

$$|m_{1,2}|^2 = 2k^2 + \frac{A^2}{8}|\epsilon|^2 \pm \sqrt{\left(2k^2 + \frac{A^2}{8}|\epsilon|^2\right)^2 - 4k^4}, \quad |m_3|^2 \approx 4A^2, \quad (3.13)$$

thus

$$r_\Delta = \frac{|\epsilon|^2}{16 \cos 2\theta_{12}}. \quad (3.14)$$

If  $\epsilon$  is real, we obtain

$$\tan 2\theta_{23} = \frac{2}{\epsilon} \quad (3.15)$$

that results in the following relations among the observables:

$$\tan^2 2\theta_{23} = \frac{1}{4r_\Delta \cos 2\theta_{12}}. \quad (3.16)$$

From (3.16) we obtain a deviation from the maximal mixing

$$\frac{1}{2} - \sin^2 \theta_{23} = \left[ \frac{1}{r_\Delta \cos 2\theta_{12}} + 4 \right]^{-1/2} \approx 0.1. \quad (3.17)$$

Rewriting  $\sin \theta_{13}$  in (3.12) in terms of the mixing parameter, we find

$$\sin \theta_{13} = -\frac{\tan 2\theta_{12}}{8 \tan^2 2\theta_{23}} = -\frac{1}{2} \sin 2\theta_{12} r_\Delta, \quad (3.18)$$

that coincides with (3.5) for  $\cot \theta_{23} \approx 1$ .

Establishing small ( $\sim r_\Delta$ ) 1-3 mixing and a relatively large deviation of 2-3 mixing from maximal:  $\sin^2 2\theta_{23} \approx 0.96$ , will be in support of the considered possibility. Such a deviation from maximal mixing can be tested by future long baseline experiments [12, 13]. These are typical features of a mass matrix whose  $\nu_\mu - \nu_\tau$  symmetry is broken in the  $\nu_\mu - \nu_\tau$  block.

If  $\epsilon$  is complex,  $\epsilon = |\epsilon| \exp(i\phi_\epsilon)$ , the Majorana CP-violating phases will be nonzero. Let us evaluate these phases. According to our convention  $m_1$  is real, thus from the condition  $(m_\nu)_{ee} = 0$  we obtain

$$\text{Im}(m_2)s_{12}^2 + \text{Im}(m_3)s_{13}^2 = 0, \quad (3.19)$$

or using eqs. (3.12), (3.13)

$$\sin \phi_2 \approx \sin \phi_3 (r_\Delta)^{3/2}. \quad (3.20)$$

Thus, in the first approximation the phase of  $m_2$  can be neglected. To obtain the phase of  $m_3$ , we use the rephasing invariant  $I_3$

$$\frac{m_{\tau\tau} m_{\mu\mu}}{m_{\mu\tau}^2} = 1 + \epsilon.$$

Since the 1-3 mixing is small, the  $I_3$  invariant can be rewritten as

$$\frac{(\bar{m}_2 c_{23}^2 + m_3 s_{23}^2)(\bar{m}_2 s_{23}^2 + m_3 c_{23}^2)}{(\bar{m}_2 - m_3)^2 s_{23}^2 c_{23}^2} = 1 + \epsilon, \quad (3.21)$$

where

$$\bar{m}_2 \approx m_2 c_{12}^2 + m_1 s_{12}^2.$$

Notice that  $\bar{m}_2$  is real in our approximation. From (3.21) we obtain

$$\frac{\bar{m}_2 m_3}{(\bar{m}_2 - m_3)^2 s_{23}^2 c_{23}^2} = \epsilon, \quad (3.22)$$

and since  $\bar{m}_2 \ll m_3$

$$\phi_3 \simeq -\phi_\epsilon.$$

So, the phase of  $m_3$  is equal to the phase of  $\epsilon$  and in general can be large.

2.  $h_{\tau e} = 0$ : This equality results in zero  $\delta$  and

$$\sin \theta_{13} \approx 0.5 \sin 2\theta_{12} \tan \theta_{23} r_\Delta, \quad (3.23)$$

or  $\delta = \pi$  and  $\sin \theta_{13} = -0.5 \sin 2\theta_{12} \tan \theta_{23} r_\Delta$ . The equality  $h_{\tau e} = 0$  means that

$$m_{e\mu} = -\frac{m_{ee} m_{e\tau}^* + m_{e\tau} m_{\tau\tau}^*}{m_{\mu\tau}^*}, \quad (3.24)$$

and in the case of normal hierarchical scheme this leads to the mass matrix

$$m_\nu = \begin{bmatrix} g' & -\frac{g' k'^* + k' A^*}{A^*(1+\epsilon'_2)} & k' \\ \dots & A(1+\epsilon'_3) & A(1+\epsilon'_2) \\ \dots & \dots & A \end{bmatrix}. \quad (3.25)$$

If  $g' = \epsilon'_2 = 0$ , we obtain

$$m_\nu = \begin{bmatrix} 0 & -k' & k' \\ \dots & A(1+\epsilon') & A \\ \dots & \dots & A \end{bmatrix}. \quad (3.26)$$

The mixing angles and the splittings are the same as in the previous case except that

$$\theta_{23} = \frac{\pi}{4} + \frac{\text{Re}[\epsilon']}{4} + \frac{|\epsilon'|^2}{8}.$$

3. Equality  $h_{\mu\tau} = 0$  leads to

$$\Delta m_{31}^2 s_{23} c_{23} + \Delta m_{21}^2 (s_{23} c_{23} \cos 2\theta_{12} - s_{12} c_{12} s_{13} \cos 2\theta_{23}) = 0$$

which is not compatible with the data.

In summary, in this subsection, we have found that although  $h_{\mu\tau} = 0$  is not compatible with the data,  $h_{e\mu} = 0$  or  $h_{e\tau} = 0$  yield vanishing  $\sin \delta$  and a small but nonzero value for  $s_{13}$  of order of  $\Delta m_{21}^2 / \Delta m_{31}^2$ .

### 3.2 Small Dirac phase

According to (2.5) if all the elements of  $h_{\alpha\beta}$  are real, no CP-violating effects appear in oscillations but in general  $\theta_{13}$  and the Majorana phases can be nonzero. By straightforward but cumbersome calculations, it can be shown that to the leading order in  $\sin \theta_{13}$  and  $\cos 2\theta_{23}$ , the mass matrix which satisfies this condition can be parameterized as

$$m_\nu = \begin{bmatrix} r - 2xs - t & s(1 + \eta x - \alpha) + \eta t & -s(1 - \eta x + \alpha) + \eta t \\ \dots & r + 2\alpha t - s\eta & t \\ \dots & \dots & r - 2\alpha t + \eta s \end{bmatrix}, \quad (3.27)$$

where  $\alpha, \eta \ll x \sim 1$  are real numbers but  $r, s$  and  $t$  can in general be complex quantities. It can be shown that regardless of the mass scheme (hierarchical or degenerate; normal or inverted) the parameters of this mass matrix are immediately related to the observables as

$$\cot 2\theta_{12} = \frac{x}{\sqrt{2}}, \quad c_{23} = (1 - \alpha)/\sqrt{2}$$

and

$$\sin \theta_{13} = \eta/\sqrt{2}.$$

In order to reproduce the observed neutrino mass splitting, the complex parameters  $r, s$  and  $t$  should satisfy certain relations. It can be shown that

$$\begin{aligned} m_1 &= \frac{-2(r-t)s_{12}^2 - 2xsc_{12}^2}{\cos 2\theta_{12}} \\ m_2 &= \frac{2(r-t)c_{12}^2 + 2xss_{12}^2}{\cos 2\theta_{12}} \\ m_3 &= r + t. \end{aligned} \quad (3.28)$$

Thus,

$$\Delta m_{21}^2 = \frac{4(|r-t|^2 - |s|^2 x^2)}{\cos 2\theta_{12}}.$$

The normal mass hierarchy requires  $|r-t|^2, x^2|s|^2 \ll |r+t|^2$ ; while, in order to achieve inverted hierarchy, one needs  $|r+t|^2, (|r-t|^2 - x^2|s|^2) \ll (|r-t|^2 + x^2|s|^2)$ .

For the mass matrix given in eq. (3.27) with general but small  $\alpha$  and  $\eta$  the Dirac phase is small but nonzero:  $\delta \sim \mathcal{O}(\text{Max}[\eta, \alpha]) \ll 1$ . In the specific case that

$$\eta^2 + 2\eta\alpha x - 2\alpha^2 = 0, \quad (3.29)$$

the Dirac phase vanishes, exactly. This can be verified in two steps: 1) If the above relation holds,  $h_{e\mu}$ ,  $h_{e\tau}$  and  $h_{\mu\tau}$  are all real which means the Jarlskog invariant is zero; 2) using the criterion (3.1), we find that  $s_{13} \neq 0$  which means  $\sin \delta$  must vanish.

The cases discussed in the previous section ( $h_{e\mu}, h_{e\tau} = 0$ ) can be considered as special instances of the mass matrix with the form in eq. (3.27), provided that (3.29) is satisfied. For example,

$$\eta \simeq \pm \frac{4}{x} \frac{|r-t|^2 - x^2 |s|^2}{|r+t|^2 - 4|(r-t)s_{12}^2 + xsc_{12}^2|^2 / \cos^2 2\theta_{12}} \quad (3.30)$$

yields vanishing  $h_{e\mu}$ . Using (3.28) and remembering that  $\eta = \sqrt{2} \sin \theta_{13}$ , it can be confirmed that (3.30) yields the same relation between  $\sin \theta_{13}$  and the mass splittings that we expected in the case of  $h_{e\mu} = 0$ ; *i.e.*, eq. (3.30) corresponds to (3.18).

In what follows we show that by a change of basis, matrix (3.27) acquires a simple form which will be easier to incorporate in models. Let us define  $(\tilde{\nu}_0 \tilde{\nu}_+ \tilde{\nu}_-) \equiv (\nu_e \nu_\mu \nu_\tau) V_b^T$ , where  $V_b$  is a unitary matrix which neglecting  $O(\eta^2, \alpha^2)$ , can be written as

$$V_b = \begin{bmatrix} i & -i\eta/2 & -i\eta/2 \\ \eta/\sqrt{2} & (1+\alpha)/\sqrt{2} & (1-\alpha)/\sqrt{2} \\ 0 & i(1-\alpha)/\sqrt{2} & -i(1+\alpha)/\sqrt{2} \end{bmatrix}. \quad (3.31)$$

It is straightforward to show that in this basis, up to  $O(\eta^2, \alpha^2)$ , matrix (3.27) obtains the following simple form

$$\begin{bmatrix} -r+t+2xs & 0 & -\sqrt{2}s \\ 0 & r+t & 0 \\ -\sqrt{2}s & 0 & -r+t \end{bmatrix}. \quad (3.32)$$

In this subsection, we have derived the general form of a neutrino mass matrix for which  $|\sin \delta| \ll 1$ . We have then shown that by changing the basis, this matrix acquires a simple form which will be easy to incorporate into models.

### 3.3 Zero Dirac phase with nonzero elements of $h_{\alpha\beta}$

Consider a matrix  $h$  with vanishing Jarlskog invariant but non-zero off-diagonal elements. In general the equality  $J_{CP} = 0$  is equivalent to the condition that by rephasing the neutrino fields the  $h$  matrix can be made real. However, because of the possibility of nonzero Majorana phases,  $J_{CP} = 0$  does not necessarily mean that  $m_\nu$  can be made real by rephasing. In this section we formulate the necessary conditions on  $m_\nu$  for  $J_{CP} = 0$  in a specific class of mass patterns, demonstrating how the invariants defined in the previous section can simplify the analysis.

Let us consider a matrix for which  $I_3, I_4$  and  $I_5$  are all real. After a proper rephasing, the general form of such a matrix can be written as

$$m_\nu = \begin{bmatrix} u & v & nv \\ \dots & B & lB \\ \dots & \dots & (1+\kappa)B \end{bmatrix}, \quad (3.33)$$

where  $l$ ,  $n$  and  $\kappa$  are real numbers but  $u$ ,  $v$  and  $B$  can have complex values. This assumption means that the phases of  $m_{e\mu}$  and  $m_{e\tau}$  are equal, and also all the elements of  $\mu - \tau$  block have the same phase. In this case by rephasing the neutrino fields one can eliminate the phases from all the elements but  $m_{ee}$ ; *i.e.*, only  $u$  remains complex. Apparently, if  $m_{ee} = 0$ , there will be no CP-violation in the lepton sector.

Matrix (3.33) automatically gives

$$\text{Im}[h_{\mu\tau}] = 0.$$

The two other elements are equal to

$$\begin{aligned} h_{e\mu} &= uv^* + vB^*(1 + ln), \\ h_{\tau e} &= nvu^* + Bv^*[l + n(1 + \kappa)]. \end{aligned}$$

Then the condition of the absence of the CP violation in oscillations,  $\mathcal{J} = 0$ , reduces to

$$\text{Im}[uB(v^*)^2](l + n(1 + \kappa)) - n(1 + ln) = 0. \tag{3.34}$$

Let us consider solutions of this equation and their implications.

1. The condition (3.34) is satisfied if  $u = 0$ , as we have noticed before. Two other trivial solutions,  $B = 0$  and  $v = 0$  are ruled out by the neutrino data.
2. Another possible solution (again for arbitrary  $l$ ,  $n$  and  $\kappa$ ) is

$$\text{Im}[I_1^*] = \text{Im}[uB(v^*)^2] = 0. \tag{3.35}$$

For two other invariants we obtain the following:  $I_2 = n(1 + \kappa)v^2u^*B^*$  - that has the same imaginary part as  $I_1$ , and  $I_5 = ln(1 + \kappa)|B|^2$  which is real. Therefore if condition (3.35) is satisfied, all the invariants,  $I_1, I_2, I_5$  will be real and, according to the general consideration of section 2, there will be no CP-violation even for nonzero  $u$ ; *i.e.*, all the physical phases will vanish.

The same conclusion can be obtained in a different way. As we have already pointed out, we can make all the parameters of matrix (3.33) except  $u$  real by rephasing. In this case, condition (3.35) means that  $u$  should be real too. Notice that the 1-3 mixing is, in general, nonzero.

3. The last non-trivial solution of (3.34) is

$$l + n(1 + \kappa) = n(1 + ln). \tag{3.36}$$

Performing diagonalization of the mass matrix and remembering that the 1-2 mixing is nonzero, it is straightforward to check that the condition (3.36) leads to a vanishing 1-3 mixing. So,  $\mathcal{J} = 0$  is satisfied trivially due to the zero mixing.

Thus, with a matrix of form (3.33), the Jarlskog invariant can vanish if and only if either the lepton sector is CP-invariant (*i.e.*, both the Dirac and Majorana phases vanish) or  $\sin \theta_{13} = 0$ . In other words, for (3.33) we cannot have  $\sin \theta_{13} \neq 0$ ,  $\sin \delta = 0$ .

#### 4. Maximal CP violating phase

Several neutrino mass matrix textures have been proposed that predict a maximal value for the Dirac CP-violating phase [3, 4, 15, 16]. In other words, for given values of mixing angles, they predict the Jarlskog invariant to be maximal which implies  $|\sin \delta| = 1$ . For the experiments proposed to directly search for the CP-violation in the neutrino oscillations, this means that for given values of mixing angles, the asymmetry  $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  is maximal.

It has been suggested in [17] to use astrophysical neutrinos to determine the value of  $\delta$  without directly measuring the CP-violating effects. For stable neutrinos, the effect of  $\delta$  will be too small to resolve. However, if at cosmological distances the heavier neutrinos decay into  $\nu_1$ , the ratio of neutrino fluxes of different flavors will be sensitive to  $\sin \theta_{13} \cos \delta$ :

$$\Phi_{\nu_e} : \Phi_{\nu_\mu} : \Phi_{\nu_\tau} = |U_{e1}|^2 : |U_{\mu1}|^2 : |U_{\tau1}|^2,$$

which for  $c_{23}^2 = s_{23}^2 = 1/2$  and  $s_{13} \ll 1$  can be written as

$$|U_{e1}|^2 : |U_{\mu1}|^2 : |U_{\tau1}|^2 \simeq c_{12}^2 : \frac{s_{12}^2 + 2s_{12}c_{12}s_{13} \cos \delta}{2} : \frac{s_{12}^2 - 2s_{12}c_{12}s_{13} \cos \delta}{2}.$$

In the case of the maximal CP-violating phase, like in the case of  $s_{13} = 0$ , we expect  $\Phi_{\nu_\mu}/\Phi_{\nu_e} = \tan^2 \theta_{12}/2$ , which can be in principle checked by Icecube [17, 18]. Thus, from the point of view of the indirect measurements, the maximal CP-violation value of the phase ( $\cos \delta = 0$ ) is also special.

As noticed in [19, 20], the maximal Dirac CP-violating phase can be explained by the so-called  $\mu - \tau$  reflection symmetry under which  $\nu_\mu$  and  $\nu_\tau$  transform into the CP-conjugate of each other. We first study various new aspects of this symmetry, and then in section 5, generalize it to accommodate non-maximal values of  $\delta$  as well as a deviation of the 2-3 mixing from  $\pi/4$ .

##### 4.1 $\mu - \tau$ reflection symmetry

The  $\mu - \tau$  reflection [19, 20] is defined as follows

$$\nu_e \rightarrow \xi_1 \nu_e^c, \quad \nu_\mu \rightarrow \xi_2 \nu_\tau^c, \quad \nu_\tau \rightarrow \xi_3 \nu_\mu^c, \tag{4.1}$$

where  $\xi_i$  are phase factors. These transformations are combinations of charge conjugation, parity and permutation in the flavor space. Notice that if  $\xi_2 \neq \xi_3$ ,  $m_{e\mu}$  and  $m_{e\tau}$  should both vanish which contradicts the observations. Thus, we should take

$$\xi_2 = \xi_3. \tag{4.2}$$

The most general form of neutrino mass matrix invariant under the  $\mu - \tau$  reflection transformations is

$$m_\nu = \begin{bmatrix} if\xi_1^* & we^{-i\sigma} & -w\xi_1^*\xi_2^*e^{i\sigma} \\ \dots & ye^{2i\beta} & -iz\xi_2^* \\ \dots & \dots & -ye^{-2i\beta}(\xi_2^*)^2 \end{bmatrix}, \dots \tag{4.3}$$

where  $w, y, z, f$  and  $\sigma$  are all real. By rephasing  $\nu_\mu \rightarrow e^{-i\beta}\nu_\mu$ ,  $\nu_\tau \rightarrow i\xi_2 e^{i\beta}\nu_\tau$ ,  $\nu_e \rightarrow \sqrt{-i\xi_1}\nu_e$  and redefining  $\sigma$ , the mass matrix obtains the form<sup>2</sup>

$$m_\nu = \begin{bmatrix} f & we^{-i\sigma} & -we^{i\sigma} \\ \dots & y & z \\ \dots & \dots & y \end{bmatrix}. \quad (4.4)$$

For such a mass matrix,  $\mathcal{J}$  [see eq. (2.6)] is equal to

$$\mathcal{J} = -2w^2y \sin 2\sigma [w^2(-z + f) + z(f - z)^2 - y^2z + yw^2 \cos 2\sigma]$$

which, in general, is nonzero for  $\sin 2\sigma, w, y \neq 0$ . In the following we show that for  $\sin 2\sigma \neq 0$  the matrix (4.4) implies a nonzero 1-3 mixing and  $\delta = \pi/2$ .

Eq. (4.4) yields the following relations among the elements of the matrix  $h$ :

$$h_{\mu\mu} = h_{\tau\tau} \quad [= w^2 + y^2 + z^2], \quad (4.5)$$

and

$$h_{e\mu} = -h_{\tau e} = -h_{e\tau}^* \quad [= w(fe^{i\sigma} - ze^{i\sigma} + ye^{-i\sigma})]. \quad (4.6)$$

Using explicit expressions for  $h_{\alpha\beta}$  in terms of the oscillation parameters we find from eq. (4.5)

$$\cos 2\theta_{23} = -4\rho_\Delta s_{12}c_{12}s_{23}c_{23}s_{13} \cos \delta, \quad (4.7)$$

where

$$\rho_\Delta = \frac{\Delta m_{21}^2}{\Delta m_{31}^2 c_{13}^2 + (s_{12}^2 s_{13}^2 - c_{12}^2) \Delta m_{21}^2}.$$

From eq. (4.6),  $|h_{e\mu}| = |h_{\tau e}|$ , we obtain

$$\begin{aligned} & \cos 2\theta_{23} [(\Delta m_{31}^2)^2 s_{13}^2 c_{13}^2 - (\Delta m_{21}^2)^2 s_{12}^2 c_{12}^2 c_{13}^2] \\ & - 4\Delta m_{31}^2 \Delta m_{21}^2 s_{13} c_{13}^2 s_{23} c_{23} s_{12} c_{12} \cos \delta = 0. \end{aligned} \quad (4.8)$$

For the observed values of mass squared differences and mixing angles, equalities (4.7) and (4.8) are satisfied if  $\cos 2\theta_{23} = 0$  (*i.e.*, 2-3 mixing is maximal) and

$$\sin \theta_{13} \cos \delta = 0. \quad (4.9)$$

For  $\sin \theta_{13} = 0$  and  $\cos 2\theta_{23} = 0$  the rephasing invariants  $I_1$  and  $I_2$  (2.10) are equal, which in the case of (4.4), is realized provided that  $e^{4i\sigma} = 1$ . Notice that for  $\sin \theta_{13} = 0$ , or in other words for  $\sigma = 0, \pi, \pm\pi/2$ , the  $\mu - \tau$  symmetry [21] is restored and the CP in oscillations is conserved. For a general value of  $\sigma$  (not belonging to  $\{0, \pm\pi/2, \pi\}$ ),  $\sin \theta_{13}$  is nonzero and eq. (4.9) can be satisfied only if  $\cos \delta = 0$ .

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<sup>2</sup>A similar texture has been considered in [4, 15, 16] where the dominant part of the mass matrix is taken to be symmetric under the  $\mu - \tau$  exchange and the nonzero phases are introduced as parameters that break the  $\mu - \tau$  symmetry. In contrast, the  $\mu - \tau$  reflection symmetry, even if exact, can accommodate nonzero phases.

In addition to predicting maximal values for  $\theta_{23}$  and  $\delta$ , the  $\mu - \tau$  reflection symmetry predicts zero (or equal to  $\pi$ ) Majorana phases. This can be proved by explicitly diagonalizing the matrix  $m_\nu$ . This conclusion holds for any mass scheme: normal/inverted, hierarchical/degenerate.

The  $\mu - \tau$  reflection symmetry is strongly broken in the charged lepton sector. As a result, we expect the radiative corrections induced by the charged lepton sector to break this symmetry in the neutrino sector, too (see section 4.3).

In what follows, we study the normal mass hierarchy in the context of the  $\mu - \tau$  reflection symmetry. In order to reproduce the normal hierarchy, the elements of (4.4) should satisfy:

$$|y^2 - z^2| \sim w^2, f^2 \ll y^2, z^2.$$

Using these inequalities it is straightforward to show that

$$\tan \theta_{23} = 1, \quad \tan 2\theta_{12} \simeq \frac{2\sqrt{2}w \cos \sigma}{y - z - f}, \quad \sin \theta_{13} \simeq \frac{\sqrt{2}w \sin \sigma}{y + z}, \quad (4.10)$$

and

$$m_3 = y + z, \quad m_{1,2} = \frac{y - z - f \pm \sqrt{(z - y - f)^2 + 8w^2 \cos^2 \sigma}}{2} \quad (4.11)$$

and finally, as expected,  $\delta = \pi/2$  but the Majorana phases vanish. Combining the above formulas and the information on masses and mixing we obtain  $w \cos \sigma / (y + z) \sim 0.1$ . For  $\tan \sigma \sim 1$ ,  $\theta_{13}$  can saturate its upper bound. Inversely, the bound on  $\sin \theta_{13}$  can be interpreted as an upper bound on  $\tan \sigma$ . Notice that for  $\sin \sigma \rightarrow 0$ , the  $\mu - \tau$  exchange symmetry is restored and as a result  $\sin \theta_{13} = 0$ , and consequently, there will be no CP-violating effects.

Now let us consider the possibility to reproduce matrix (4.4) for the normal mass hierarchy by the seesaw mechanism. Without loss of generality we choose the mass basis for the right-handed neutrinos. Suppose that under the  $\mu - \tau$  reflection symmetry, the right-handed neutrinos transform into the charge conjugates of themselves,  $N_i \rightarrow N_i^c$ . In order for the right-handed neutrino mass matrix to be symmetric under the  $\mu - \tau$  reflection, it should be real:

$$\mathcal{L}_{\text{mass}} = - \sum_{i=1}^3 M_i N_i^T C N_i \quad \text{with} \quad M_i = M_i^*.$$

Moreover, the  $\mu - \tau$  reflection symmetry with  $N_i \rightarrow N_i^c$  implies the following form for the neutrino Yukawa couplings

$$Y_\nu = \begin{bmatrix} a_1 & b_1 e^{i\kappa_1} & b_1 e^{-i\kappa_1} \\ a_2 & b_2 e^{i\kappa_2} & b_2 e^{-i\kappa_2} \\ a_3 & b_3 e^{i\kappa_3} & b_3 e^{-i\kappa_3} \end{bmatrix} \quad (4.12)$$

with real  $a_i$  and  $b_i$ .

We assume that the right-handed neutrino  $N_3$  dominates in generation of the light mass matrix (single right-handed neutrino dominance [22]). This implies  $a_1^2, |b_1|^2, |b_2|^2 \ll$



$|b_3|^2 M_1/M_3$ . Moreover, suppose the dominant part of the Lagrangian that involves  $N_3$  preserves  $L_e$ ; *i.e.*,  $a_3 = 0$ . Then, the mass matrix of the light neutrinos will be equal to

$$v^2 \begin{bmatrix} \frac{a_1^2 + (a_1^*)^2}{M_1} & \frac{a_1 b_1 + a_1^* b_2}{M_1} & \frac{a_1 b_2^* + a_1^* b_1^*}{M_1} \\ \dots & \frac{b_3^2}{M_3} + \frac{b_2^2 + b_1^2}{M_1} & \frac{|b_3|^2}{M_3} + \frac{b_2 b_1^* + b_1 b_2^*}{M_1} \\ \dots & \dots & \frac{(b_3^*)^2}{M_3} + \frac{(b_2^*)^2 + (b_1^*)^2}{M_1} \end{bmatrix} \quad (4.13)$$

which, considering the realness of  $M_i$ , is precisely of form (4.4).

## 4.2 $\mu - \tau$ exchange versus $\mu - \tau$ reflection symmetry

Let us compare the phenomenological consequences of the  $\mu - \tau$  reflection and exchange symmetries and discuss the necessary and sufficient conditions for having maximal CP-violating phases. As follows from the discussion in section (4.1) both symmetries guarantee that

$$h_{\mu\mu} = h_{\tau\tau}, \quad |h_{e\mu}| = |h_{e\tau}|. \quad (4.14)$$

As shown in section (4.1), the above equalities, in turn, lead to  $\cos 2\theta_{23} = 0$  (maximal 2-3 mixing) and  $\sin \theta_{13} \cos \delta = 0$ . However, realization of the last equality is different for the two symmetries: while  $\nu_\mu \leftrightarrow \nu_\tau$  implies  $\sin \theta_{13} = 0$ , nonzero  $\sin \theta_{13}$  is compatible with the  $\mu - \tau$  reflection symmetry, and as a result, the latter symmetry requires that  $\cos \delta = 0$ .

Now, let us discuss whether equalities  $\cos 2\theta_{23} = \sin \theta_{13} \cos \delta = 0$  necessarily imply the aforementioned symmetries. It is well-known (and in fact, straightforward to confirm) that the equalities

$$\cos 2\theta_{23} = \sin \theta_{13} = 0 \quad (4.15)$$

are compatible with neutrino data (*i.e.*, nonzero  $\sin 2\theta_{12}$  and  $\Delta m_{21}^2$ ) only if there is a  $\mu - \tau$  symmetry. In other words, (4.15) implies  $I_1 = I_2$  [see eq. (2.10)] and  $|m_{\tau\tau}| = |m_{\mu\mu}|$ . On the contrary, to have

$$\cos 2\theta_{23} = \cos \delta = 0 \quad (4.16)$$

the  $\mu - \tau$  reflection symmetry is not a necessary condition. Indeed, the two conditions  $h_{\mu\mu} = h_{\tau\tau}$  and  $|h_{e\mu}| = |h_{e\tau}|$  can be simultaneously satisfied while  $|m_{e\mu}| \neq |m_{e\tau}|$ . To show this, let us write the elements of  $m_\nu$  in terms of oscillation parameters:

$$\begin{aligned} m_{e\mu} &= s_{12}c_{12}c_{23}c_{13}(|m_2|e^{i\phi_2} - |m_1|) \\ &\quad + s_{23}s_{13}c_{13}(|m_3|e^{i\phi_3}e^{-i\delta} - |m_1|c_{12}^2e^{i\phi_1}e^{i\delta} - |m_2|s_{12}^2e^{i\phi_2}e^{i\delta}), \\ m_{e\tau} &= -s_{12}c_{12}s_{23}c_{13}(|m_2|e^{i\phi_2} - |m_1|e^{i\phi_1}) \\ &\quad + c_{23}s_{13}c_{13}(|m_3|e^{i\phi_3}e^{-i\delta} - |m_1|c_{12}^2e^{i\phi_1}e^{i\delta} - |m_2|s_{12}^2e^{i\phi_2}e^{i\delta}). \end{aligned}$$

Apparently for  $|m_{e\mu}| = |m_{e\tau}|$ , in addition to equalities (4.15), the sines of the Majorana phases must also vanish. However, for general nonzero phases,  $(|m_{e\mu}| - |m_{e\tau}|) / (|m_{e\mu}| + |m_{e\tau}|) \sim 1$ . The maximal 2-3 mixing and Dirac phase do not guarantee that the  $\mu - \tau$  reflection symmetry exists. On the other hand, the  $\mu - \tau$  reflection symmetry is not necessary for maximal CP-violating phase.

Summarizing, zero Majorana phases and eq. (4.16) imply  $I_1 = I_2^* \neq I_2$  and  $I_3 = I_3^*$  which (by appropriate rephasing) results in matrix (4.4). However, if Majorana phases are nonzero even for (4.16), the element  $|m_{e\mu}|$  can be different from  $|m_{e\tau}|$ . Therefore, to prove the existence of a  $\mu - \tau$  reflection symmetry not only confirming (4.16) is necessary but one has to also show that the Majorana phases vanish.

Notice that the phases of off-diagonal elements of  $h$  are not rephasing invariant and are not therefore physical. As a result, despite the claim in [4],  $h_{e\tau} = -\text{sgn}[s_{23}]h_{e\mu}^*$  does not guarantee that  $\delta = \pi/2$ ,  $\cos \theta_{23} = 1$ . In fact, for any pair of  $(\delta, \theta_{23})$  satisfying eq. (4.8), we can have  $|h_{e\mu}| = |h_{e\tau}|$ . Then, by appropriate rephasing of the fields the condition  $h_{e\tau} = -\text{sgn}[s_{23}]h_{e\mu}^*$  can be satisfied.

### 4.3 Renormalization group effects

The CP-violating phase is not invariant under renormalization. Furthermore, the physics responsible for a certain pattern of the mass matrix can become manifest only at very high scales, *e.g.*, the Grand unification scale  $M_{GU}$ . The RG effects should be therefore taken into account when confronting with the low energy observations.

The radiative corrections change the matrix and break the  $\mu - \tau$  reflection symmetry, so the value of  $\delta$  is expected to be modified. For illustration let us consider the running of the effective  $D = 5$  operator

$$|H|^2 \sum_{ij} \nu_i^T \frac{(m_\nu)_{ij}}{\langle H \rangle^2} C \nu_j$$

from the scale of decoupling of the heavy neutrinos (the seesaw scale) down to the low energies. Neglecting the corrections proportional to  $m_\mu^2/\langle H \rangle^2$  and  $m_e^2/\langle H \rangle^2$  and absorbing the flavor-independent corrections in the definition of the overall mass scale we find that matrix (4.4) will modify into

$$m_\nu = \begin{bmatrix} f & we^{-i\sigma} & -we^{i\sigma}(1 + \tilde{\epsilon}) \\ \dots & y & z(1 + \tilde{\epsilon}) \\ \dots & \dots & y(1 + 2\tilde{\epsilon}) \end{bmatrix}. \tag{4.17}$$

In the standard model, we have  $\tilde{\epsilon} \sim \frac{\log[\Lambda/M_Z]}{16\pi^2} \frac{m_\tau^2}{\langle H \rangle^2}$ , which setting the cutoff  $\Lambda = 10^{12}$  GeV, is of order of  $10^{-5}$ . In the supersymmetric version with large  $\tan \beta$  the corrections can be much larger:  $\tilde{\epsilon} \sim 10^{-2}$ .

As a result of the corrections, both  $\theta_{23}$  and  $\delta$  are shifted from their maximal mixing and CP-violation values by a tiny amount. It can be shown that

$$\cos \theta_{23} - \frac{1}{\sqrt{2}} = \frac{2y^2 + w^2}{4\sqrt{2}yz} \tilde{\epsilon},$$

and for normal mass hierarchy ( $w^2 \ll y^2 \simeq z^2$ ) the deviation from maximal mixing is approximately equal to  $\tilde{\epsilon}/2\sqrt{2}$ . The formula for the deviation of  $\delta$  from  $\pi/2$  is more complicated. In the case of normal hierarchy, we obtain

$$\cos \delta \sim \tilde{\epsilon} \left( \frac{\Delta m_{31}^2}{\Delta m_{21}^2} \right)^{1/2} \ll 1.$$

#### 4.4 $A_4$ symmetry and maximal CP violation

The  $\mu - \tau$  reflection symmetry can be implicit or accidental in models with certain flavor symmetries. We show this using the example of a specific model based on the  $A_4$  symmetry. It has been observed that in models based on  $A_4$ , the Dirac phase is maximal [23]. The neutrino mass matrix in the flavor basis is given by

$$m_\nu = f^2 \langle H \rangle^2 U_L^T M_N^{-1} U_L, \tag{4.18}$$

where

$$U_L = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{2i\pi/3} & e^{-2i\pi/3} \\ 1 & e^{-2i\pi/3} & e^{2i\pi/3} \end{bmatrix}, \tag{4.19}$$

and  $f$  is the Yukawa coupling, and in the symmetry basis the Dirac mass matrix is proportional to the unit matrix.

In the limit of exact  $A_4$  one has  $M_N = \text{Diag}(M, M, M)$ , and as is shown in [23], the corresponding light neutrino mass matrix,

$$m_\nu = \frac{f^2 \langle H \rangle^2}{M} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{4.20}$$

leads to a degenerate mass spectrum. Deviations of  $M_N$  from being proportional to the unit matrix, which can take place due to the soft breaking of the  $A_4$  symmetry or due to the radiative corrections, can lead to phenomenologically acceptable mass matrix for light neutrinos [23].

Let us show that for *any real* (and non-singular) matrix  $M_N$ , the Dirac phase produced by  $m_\nu$  (4.18) is maximal,  $\delta = \pi/2$ . Indeed, in the flavor basis, the Dirac mass matrix of neutrinos is given by  $m_D \equiv f \langle H \rangle U_L$ . It is easy to check that in this case both the Dirac and the Majorana mass terms ( $M_N$  is real) are invariant under the  $\mu - \tau$  reflection symmetry (4.1) with

$$N_i \rightarrow N_i^c. \tag{4.21}$$

The whole neutrino sector will therefore be invariant under the  $\mu - \tau$  reflection and this, in turn, leads to the maximal Dirac phase.

### 5. Nonzero CP violating phase

In this section, we introduce a particular form of CP-flavor transformation that can be considered as a generalization of the  $\mu - \tau$  reflection transformations. We then discuss how this symmetry can be realized within the framework of the seesaw mechanism and discuss its implications for the leptogenesis.

## 5.1 Generalized $\mu - \tau$ reflection symmetry

Let us introduce the following CP-flavor transformations:

$$\nu_\alpha \rightarrow \sum_\beta P_{\alpha\beta} \nu_\beta^C, \quad (5.1)$$

where  $P$  is a unitary matrix and  $\alpha$  and  $\beta$  are flavor indices. We consider consequences of the invariance of the neutrino mass terms with respect to these transformations. Apparently (5.1) is a generalization of the  $\mu - \tau$  reflection.

Below we define  $P$  in a specific way that implies a definite value for the Dirac phase which depends on parameters of transformation (5.1). Let us consider the following symmetric form for  $P$

$$P(\theta_{23}, \phi) = U_{23}(\theta_{23}) \text{Diag}[1, 1, e^{i\phi}] U_{23}^T(\theta_{23}). \quad (5.2)$$

Notice that in the case  $e^{i\phi} = -1$ , the matrix  $P$  equals

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad (5.3)$$

which corresponds to the  $\mu - \tau$  reflection with  $\xi_1 = -\xi_2 = -\xi_3 = 1$  [see eq. (4.1)].

Since  $U_{12}$  and  $\text{Diag}[1, 1, e^{i\phi}]$  commute, it is straightforward to show that

$$P(\theta_{23}, \phi) \cdot U_{PMNS}(\theta_{23}, \theta_{12}, \theta_{13}, \delta, \phi_2, \phi_3) = U_{PMNS}(\theta_{23}, \theta_{12}, \theta_{13}, \delta + \phi, \phi_2, \phi_3 + 2\phi).$$

Thus, the equality

$$P(\tilde{\theta}_{23}, \phi) \cdot m_\nu \cdot P(\tilde{\theta}_{23}, \phi) = m_\nu^* \quad (5.4)$$

is satisfied if

$$\theta_{23} = \tilde{\theta}_{23} \quad (5.5)$$

and

$$\delta = -\frac{\phi}{2}, \quad \phi_2 = 0 \text{ or } \pi, \quad \phi_3 = -\phi. \quad (5.6)$$

Eq. (5.4) implies that the mass matrix  $m_\nu$  is invariant under the CP-flavor transformation (5.1) with  $P(\theta_{23}, \phi)$  given in (5.2) only if the CP-violating phases associated with  $m_\nu$  satisfy equalities (5.6). From (5.6) we obtain a simple relation between the Dirac and Majorana phases:

$$\phi_3 = 2\delta. \quad (5.7)$$

Notice that, here,  $\sin \theta_{13}$  can take any real value (including negative values) but  $\delta$  is restricted to  $[0, \pi]$ , so the prediction is free from any ambiguity.

Let us consider some special cases:

1. To achieve a maximal Dirac phase and a small deviation of  $\theta_{23}$  from its maximal mixing value, the transformation matrix should be

$$\hat{P}[\delta = \pi/2, \theta_{23} = \pi/4 + \tilde{\theta}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2\tilde{\theta} & -1 \\ 0 & -1 & 2\tilde{\theta} \end{bmatrix}, \quad (5.8)$$

where  $\tilde{\theta} \ll 1$ .

2. Maximal mixing,  $\theta_{23} = \pi/4$ , and a given value of the Dirac phase  $\delta$ , can be achieved with

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{e^{-2i\delta}+1}{\sqrt{2}} & \frac{e^{-2i\delta}-1}{\sqrt{2}} \\ 0 & \frac{e^{-2i\delta}-1}{\sqrt{2}} & \frac{e^{-2i\delta}+1}{\sqrt{2}} \end{bmatrix}. \quad (5.9)$$

In summary, the symmetry under the CP-flavor transformation defined in (5.2) leads to certain values of the Dirac and Majorana CP-violation phases (in the standard parametrization). The phases are related to the parameter of transformation and, depending on the value of this parameter, can take any value from zero to the maximal CP-violating phase.

## 5.2 Seesaw mechanism and leptogenesis

Let us consider the implications of the neutrino mass matrices with the CP-flavor symmetry for the seesaw mechanism and leptogenesis. We introduce three heavy right-handed neutrinos  $N_i$ , and consider a basis in which the mass matrix of these neutrinos is diagonal:  $M = \text{Diag}(M_1, M_2, M_3)$ . As discussed in section (4.1), if the right-handed neutrinos transform under the  $\mu - \tau$  reflection without any flavor change,  $N_i \rightarrow N_i^c$ ,  $M_i$  should be real and the Yukawa coupling matrix should have form (4.12) with real  $a_i$  and  $b_i$ . For  $Y_\nu$  given in eq. (4.12)  $Y_\nu \cdot Y_\nu^\dagger$  is a real matrix. This conclusion is valid also for the generalized  $\mu - \tau$  reflection with a more complicated transformation matrix  $P$ : Invariance under  $N_i \rightarrow N_i^c$  and  $\nu \rightarrow P\nu^c$  implies  $Y_\nu^* = Y_\nu \cdot \hat{P}$ . As a result,

$$Y_\nu \cdot Y_\nu^\dagger = Y_\nu \hat{P} \cdot \hat{P}^\dagger Y_\nu^\dagger = Y_\nu^* \cdot Y_\nu^T.$$

Leptogenesis is driven by a combination of the Yukawa couplings known as the total asymmetry ( $\varepsilon_1$ ) which is given by  $\text{Im}[(Y_\nu \cdot Y_\nu^\dagger)_{\beta 1}^2]$ . Obviously, the total asymmetry vanishes for a model which is symmetric under the  $\mu - \tau$  reflection and, without flavor effects associated with the charged leptons, leptogenesis cannot take place in the symmetry limit. This result is in accord with [20].

However, as recently shown, for  $M_1 < 10^{13}$  GeV flavor effects can alter the situation; *i.e.*, as long as the partial asymmetry

$$\varepsilon_1^\alpha \equiv [\Gamma(N_1 \rightarrow \ell_\alpha H) - \Gamma(N_1 \rightarrow \bar{\ell}_\alpha \bar{H})] / \Gamma_{\text{tot}} \quad (5.10)$$

does not vanish, decoherence caused by charged lepton Yukawa couplings of  $\tau$  can lead to a successful leptogenesis even if  $\varepsilon_1 = 0$  [24]. In general, the mass matrix in eq. (4.12) yields nonzero  $\varepsilon_1^\tau$  which for  $M_1 < 10^{13}$  GeV can reopen the possibility of a successful leptogenesis.

Another possibility is to define the transformation of the  $N_i$  in a way that includes a flavor permutation:

$$N_1 \leftrightarrow N_2^c, \quad N_3 \rightarrow N_3^c. \quad (5.11)$$

Invariance under these transformations imply

$$Y_\nu = \begin{bmatrix} a_1 & b_1 & b_2^* \\ a_1^* & b_2 & b_1^* \\ a_3 & b_3 & b_3^* \end{bmatrix}, \quad (5.12)$$

where  $a_3$  is real but the rest of the parameters can be complex. Now the matrix  $Y_\nu \cdot Y_\nu^\dagger$  has complex off-diagonal entries which opens a possibility for leptogenesis. Moreover the transformations defined in (5.11) imply

$$M_1 = M_2.$$

This degeneracy can be slightly lifted by some additional physics [25] (notice that the  $\mu - \tau$  reflection symmetry is broken anyway in the charged lepton sector). The quasi-degeneracy of these two mass eigenstates can lead to the resonance leptogenesis.

## 6. Possible relations between the phases of the CKM and PMNS matrices

In view of strong differences between the mass and mixing patterns of leptons and quarks, one does not expect  $\delta_{\text{CKM}}$  and  $\delta$  to be equal. However, conditions can be formulated that lead to simple and immediate relations of the phases. For this purpose, we first make the following assumptions:

1. The seesaw type-I mechanism generates the neutrino masses, and therefore the light neutrino mass matrix in the flavor basis is equal to

$$m_\nu = U_L^* m_D^{\text{diag}} M_N^{-1} m_D^{\text{diag}} U_L^\dagger; \tag{6.1}$$

2. Due to the quark-lepton symmetry or unification

$$U_L = V_{\text{CKM}}^\dagger; \tag{6.2}$$

3. The matrix  $m_D^{\text{diag}} M_N^{-1} m_D^{\text{diag}}$  in (6.1) is diagonalized by a bi-maximal rotation [26]

$$U_{bm} = U_{23}^m U_{12}^m, \tag{6.3}$$

where  $U_{ij}^m$  is the maximal ( $\pi/4$ ) rotation in the  $ij$ - plane.

Then, the lepton mixing matrix will be equal to

$$U_{PMNS} = V_{\text{CKM}}^\dagger U_{bm}, \tag{6.4}$$

which leads to acceptable values for mixing angles according to the quark-lepton complementarity (QLC) scenario [27]. From (6.4) we obtain

$$|\sin \theta_{13}| = \frac{1}{\sqrt{2}} |V_{td}^\dagger + V_{cd}^\dagger| \simeq \frac{\sin \theta_C}{\sqrt{2}}. \tag{6.5}$$

This relation can be tested by forthcoming experiments such as Double CHOOZ [10], Daya Bay [11], T2K [12] and NO $\nu$ A [13]. Calculating the Jarlskog invariant and inserting the values of the mixing angles, we find

$$\sin \delta \approx \frac{|V_{ub}|}{\sin \theta_C} \sin \delta_{\text{CKM}}. \tag{6.6}$$

Here  $V_{ub}$  is an element of the CKM matrix and  $\theta_C$  is the Cabibbo angle. This leads to a suppressed value for the Dirac phase. Inserting the best fit values of  $|V_{ub}|$  and  $\sin\theta_C$  [14] in (6.6), we find  $\delta = (0.97_{-0.12}^{+0.10})^\circ$  where the uncertainty results from the relatively large uncertainty in  $\delta_{\text{CKM}}$ .

The Lagrangian of the quark sector is invariant under  $V_{\text{CKM}} \rightarrow \Gamma_\phi^\dagger V_{\text{CKM}}$ , where  $\Gamma_\phi$  is a diagonal matrix whose eigenvalues are pure phases. However,  $U_{PMNS}$  given by (6.4) changes non-trivially under this transformation. This results in an ambiguity in evaluation of  $\delta$ . In general, the seesaw mechanism can lead to the appearance of an additional phase matrix

$$U_{PMNS} = V_{\text{CKM}}^\dagger \Gamma_\phi U_{bm}. \tag{6.7}$$

Including the phase matrix  $\Gamma_\phi$ , the leptonic phase can be much larger than (6.6); however, the value of the phase should be restricted in order to make (6.7) compatible with the data on the mixing angles [28].

Let us consider another possibility that also agrees with the data. The bimaximal mixing can be generated by the charged lepton mass matrix. In this case

$$U_{PMNS} = U_{bm} V_{\text{CKM}}^\dagger, \tag{6.8}$$

which leads to

$$\sin\theta_{13} = \frac{|V_{td}^\dagger + V_{ts}^\dagger|}{\sqrt{2}} \simeq \frac{|V_{cb}|}{\sqrt{2}} \tag{6.9}$$

and

$$\sin\delta \approx -\frac{|V_{ub}|}{V_{cb}} \sin\delta_{\text{CKM}}. \tag{6.10}$$

Unfortunately, such a small value of  $\sin\theta_{13}$  will be beyond the reach of the forthcoming experiments designed to measure  $\sin\theta_{13}$  [10–12]. Thus, a positive result in these experiments will exclude this possibility.

Like in the case of (6.4), the lepton phase is suppressed:  $\delta \simeq 5^\circ$ . Thus, we conclude that without introducing new phases, the immediate relations between the quark and lepton phases lead to suppression of the leptonic CP-phase in comparison with the quark phase. Essentially, this is a consequence of the large lepton mixing.

## 7. Conclusion

We have studied symmetries, principles and phenomenological conditions which entail certain values for the Dirac CP-violating phase in the leptonic sector. Such a study gives some idea about physics behind the CP-violation as well as the implications of future measurements of the phase.

Bearing in mind that even in the quark sector, there is no theory of CP-violation, we have considered the following possibilities:

- zero (or a very small) phase;
- a maximal CP-violating phase,  $\delta = \pi/2$ ;

- an arbitrary phase which depends on the parameter of symmetry transformation;
- certain relation between the phases in the quark and lepton sectors.

By defining rephasing invariant combinations of the elements of the neutrino mass matrix, we have formulated the necessary and sufficient conditions for the zero value of the phase. In the case that all the elements of  $m_\nu$  are nonzero, CP-invariance of the mass matrix is equivalent to the realness of the three rephasing invariant combinations  $I_1$ ,  $I_2$  and  $I_5$  defined in (2.10) and (2.13). In other words, if  $I_1$ ,  $I_2$  and  $I_5$  are all real, the Dirac as well as Majorana CP-violating phases will be zero (or equal to  $\pi$ ). Particular cases in which some of the elements of  $m_\nu$  are zero have also been discussed.

We have studied the possibility that the Dirac phase is zero or very small but the Majorana phases are sizeable; i.e., CP is still broken in the lepton sector of the theory despite the vanishing Jarlskog invariant. We have derived the general form of the mass matrix that satisfies these conditions [see (3.27)]. There is no unique symmetry which leads to such a form; however, we have found that by changing the basis, the matrix can be written in a simple form [see (3.31) and (3.32)] which will be easier to incorporate in models. We have observed that the symmetries and mass patterns that lead to zero  $\delta$  also yield certain relations between the 1-3 mixing and other observables [see eqs. (3.5), (3.16), (3.23)]. These relations can be used as a test for the underlying physics.

Maximal Dirac CP-violating phase can be related to a symmetry under a specific type of combined CP and flavor (CP-flavor) transformations that is known as the  $\mu - \tau$  reflection symmetry. The symmetry leads to  $\delta = \pi/2$ , zero (or equal to  $\pi$ ) Majorana phases and maximal  $\nu_\mu - \nu_\tau$  mixing. We have shown that this symmetry is a sufficient (if mass matrix is complex) but not a necessary condition for  $\delta = \pi/2$ . In order to verify this symmetry, in addition to confirming  $\cos \delta = \cos 2\theta_{23} = 0$ , it is necessary to check that the Majorana phases are zero (or equal to  $\pi$ ).

We have proposed a generalized version of the  $\mu - \tau$  reflection symmetry. Depending on the value of the parameter of transformation, this symmetry can lead to any value of the Dirac phase. This symmetry predicts a simple relation between the Majorana and Dirac phases (5.6) and (5.7). A mass matrix symmetric under the CP-flavor transformation can be generated within the seesaw mechanism. We have discussed leptogenesis in the context of seesaw mechanism respecting the generalized  $\mu - \tau$  symmetry. If the right-handed neutrinos transform into CP-conjugate of themselves under this symmetry,  $N_i \rightarrow N_i^C$ , the total asymmetry vanishes; however, if the flavor effects are taken into account, the successful leptogenesis can still be realized. The successful leptogenesis can also be obtained in the case of non-trivial (flavor) transformation for  $N_i$ . In this case a weak violation of the symmetry can lead to the resonant leptogenesis.

The leptonic phase can be related to the quark phase in the context of quark-lepton complementarity. In this scenario, the mixing matrix in the lepton sector appears as a combination of the CKM mixing and a bi-maximal mixing. If no additional phase apart from the CKM phase is introduced, one expects a suppressed Dirac phase as a consequence of the large lepton mixing.



## A. Neutrino mass square matrix

The matrix  $h$  (2.4) can be written as

$$h = m_1^2 \text{Diag}[1, 1, 1] + U_{PMNS} \cdot \text{Diag}[0, \Delta m_{21}^2, \Delta m_{31}^2] \cdot U_{PMNS}^\dagger, \quad (\text{A.1})$$

where  $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$  and  $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$  and  $U_{PMNS}$  is the mixing matrix in the standard parametrization defined in [14]. It is straightforward to show that

$$\begin{aligned} h_{ee} &= m_1^2 + \Delta m_{21}^2 s_{12}^2 c_{13}^2 + \Delta m_{31}^2 s_{13}^2, \\ h_{\mu\mu} &= m_1^2 + \Delta m_{31}^2 s_{23}^2 c_{13}^2 + \Delta m_{21}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{13}s_{12}c_{12}s_{23}c_{23} \cos \delta), \\ h_{\tau\tau} &= m_1^2 + \Delta m_{31}^2 c_{23}^2 c_{13}^2 + \Delta m_{21}^2 (s_{23}^2 c_{12}^2 + s_{12}^2 c_{23}^2 s_{13}^2 + 2s_{12}c_{12}s_{23}c_{23}s_{13} \cos \delta). \end{aligned}$$

The absolute values of the off-diagonal elements are as follows

$$|h_{\mu\tau}| = \left| s_{23}c_{23} \left[ \Delta m_{31}^2 c_{13}^2 + \Delta m_{21}^2 \left[ (s_{12}^2 s_{13}^2 - c_{12}^2) + s_{12}c_{12}s_{13}(s_{23}^2 e^{i\delta} - c_{23}^2 e^{-i\delta}) \right] \right] \right|, \quad (\text{A.2})$$

$$|h_{e\mu}| = \left| \Delta m_{31}^2 s_{23}s_{13}c_{13}e^{-i\delta} + \Delta m_{21}^2 s_{12}c_{13} \left( c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} \right) \right|, \quad (\text{A.3})$$

$$|h_{e\tau}| = \left| \Delta m_{31}^2 c_{23}s_{13}c_{13}e^{-i\delta} + \Delta m_{21}^2 s_{12}c_{13} \left( -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{-i\delta} \right) \right|. \quad (\text{A.4})$$

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## References

- [1] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, *Evidence for the  $2\pi$  decay of the  $K_2^0$  meson*, *Phys. Rev. Lett.* **13** (1964) 138;  
BABAR collaboration, B. Aubert et al., *Observation of CP-violation in the  $B^0$  meson system*, *Phys. Rev. Lett.* **87** (2001) 091801 [[hep-ex/0107013](#)].
- [2] A. De Rujula, M.B. Gavela and P. Hernandez, *Neutrino oscillation physics with a neutrino factory*, *Nucl. Phys.* **B 547** (1999) 21 [[hep-ph/9811390](#)];  
A. Cervera et al., *Golden measurements at a neutrino factory*, *Nucl. Phys.* **B 579** (2000) 17  
*Erratum ibid.* **B 593** (2001) 731 [[hep-ph/0002108](#)];  
C. Albright et al., *Physics at a neutrino factory*, [hep-ex/0008064](#);  
K. Dick, M. Freund, M. Lindner and A. Romanino, *CP-violation in neutrino oscillations*, *Nucl. Phys.* **B 562** (1999) 29 [[hep-ph/9903308](#)];  
V.D. Barger, S. Geer, R. Raja and K. Whisnant, *Short-baseline neutrino oscillations at a neutrino factory*, *Phys. Rev.* **D 63** (2001) 033002 [[hep-ph/0007181](#)];  
M. Freund, P. Huber and M. Lindner, *Extracting matter effects, masses and mixings at a neutrino factory*, *Nucl. Phys.* **B 585** (2000) 105 [[hep-ph/0004085](#)];  
K. Hagiwara, N. Okamura and K.-i. Senda, *Physics potential of T2KK: an extension of the T2K neutrino oscillation experiment with a far detector in Korea*, [hep-ph/0607255](#).
- [3] K.S. Babu, E. Ma and J.W.F. Valle, *Underlying  $A_4$  symmetry for the neutrino mass matrix and the quark mixing matrix*, *Phys. Lett.* **B 552** (2003) 207 [[hep-ph/0206292](#)].

- [4] I. Aizawa and M. Yasue, *General property of neutrino mass matrix and CP-violation*, *Phys. Lett. B* **607** (2005) 267 [[hep-ph/0409331](#)];  
 I. Aizawa, T. Kitabayashi and M. Yasue, *Neutrino mass textures with maximal CP-violation*, *Phys. Rev. D* **72** (2005) 055014 [[hep-ph/0504172](#)]; *Determination of neutrino mass texture for maximal CP-violation*, *Nucl. Phys. B* **728** (2005) 220 [[hep-ph/0507332](#)];  
 I. Aizawa and M. Yasue, *A new type of complex neutrino mass texture and  $\mu - \tau$  symmetry*, *Phys. Rev. D* **73** (2006) 015002 [[hep-ph/0510132](#)].
- [5] K. Matsuda and H. Nishiura, *Can four-zero-texture mass matrix model reproduce the quark and lepton mixing angles and CP-violating phases?*, *Phys. Rev. D* **74** (2006) 033014 [[hep-ph/0606142](#)];  
 S. Kaneko, H. Sawanaka and M. Tanimoto, *Hybrid textures of neutrinos*, *JHEP* **08** (2005) 073 [[hep-ph/0504074](#)];  
 Z.-z. Xing, *Texture zeros and Majorana phases of the neutrino mass matrix*, *Phys. Lett. B* **530** (2002) 159 [[hep-ph/0201151](#)].
- [6] C. Jarlskog, *Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP-violation*, *Phys. Rev. Lett.* **55** (1985) 1039.
- [7] A. Kusenko and R. Shrock, *Phases in leptonic mass matrices: higher order invariants and applications to models*, [hep-ph/9403315](#).
- [8] U. Sarkar and S.K. Singh, *CP-violation in neutrino mass matrix*, [hep-ph/0608030](#).
- [9] Y. Farzan, Talk given at the XXXIII International Conference on High Energy Physics, July 26 - August 2, 2006 Moscow, Russia [http://ic hep06.jinr.ru/reports/92\\_2s3\\_15p15.farzan.pdf](http://ic hep06.jinr.ru/reports/92_2s3_15p15.farzan.pdf).
- [10] P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, *From double chooz to triple chooz: neutrino physics at the chooz reactor complex*, *JHEP* **05** (2006) 072 [[hep-ph/0601266](#)];
- [11] J. Cao, *Daya bay neutrino experiment*, *Nucl. Phys.* **155** (*Proc. Suppl.*) (2006) 229 [[hep-ex/0509041](#)];
- [12] Y. Itow et al., *The JHF-Kamioka neutrino project*, [hep-ex/0106019](#).
- [13] NOVA collaboration, D.S. Ayres et al., *NOvA proposal to build a 30-kiloton off-axis detector to study neutrino oscillations in the fermilab numi beamline*, [hep-ex/0503053](#).
- [14] PARTICLE DATA GROUP, W.-M. Yao et al., *Review of particle physics*, *J. Phys. G* **33** (2006) 1.
- [15] R.N. Mohapatra and W. Rodejohann, *Broken  $\mu - \tau$  symmetry and leptonic CP-violation*, *Phys. Rev. D* **72** (2005) 053001 [[hep-ph/0507312](#)].
- [16] Z.-z. Xing, H. Zhang and S. Zhou, *Nearly tri-bimaximal neutrino mixing and CP-violation from  $\mu - \tau$  symmetry breaking*, *Phys. Lett. B* **641** (2006) 189 [[hep-ph/0607091](#)].
- [17] Y. Farzan and A.Y. Smirnov, *Leptonic unitarity triangle and CP-violation*, *Phys. Rev. D* **65** (2002) 113001 [[hep-ph/0201105](#)];  
 J.F. Beacom, N.F. Bell, D. Hooper, S. Pakvasa and T.J. Weiler, *Sensitivity to  $\Theta_{13}$  and delta in the decaying astrophysical neutrino scenario*, *Phys. Rev. D* **69** (2004) 017303 [[hep-ph/0309267](#)];  
 W. Winter, *How astrophysical neutrino sources could be used for early measurements of neutrino mass hierarchy and leptonic CP phase*, *Phys. Rev. D* **74** (2006) 033015 [[hep-ph/0604191](#)].

- [18] J.F. Beacom, N.F. Bell, D. Hooper, S. Pakvasa and T.J. Weiler, *Measuring flavor ratios of high-energy astrophysical neutrinos*, *Phys. Rev. D* **68** (2003) 093005 [[hep-ph/0307025](#)].
- [19] P.F. Harrison and W.G. Scott,  $\mu - \tau$  reflection symmetry in lepton mixing and neutrino oscillations, *Phys. Lett. B* **547** (2002) 219 [[hep-ph/0210197](#)].
- [20] W. Grimus and L. Lavoura, *A non-standard CP transformation leading to maximal atmospheric neutrino mixing*, *Phys. Lett. B* **579** (2004) 113 [[hep-ph/0305309](#)];
- [21] T. Fukuyama and H. Nishiura, *Mass matrix of Majorana neutrinos*, [hep-ph/9702253](#);
- [22] S.F. King, *Atmospheric and solar neutrinos with a heavy singlet*, *Phys. Lett. B* **439** (1998) 350 [[hep-ph/9806440](#)].
- [23] E. Ma and G. Rajasekaran, *Softly broken  $A_4$  symmetry for nearly degenerate neutrino masses*, *Phys. Rev. D* **64** (2001) 113012 [[hep-ph/0106291](#)].
- [24] A. Abada, S. Davidson, F.-X. Josse-Michaux, M. Losada and A. Riotto, *Flavour issues in leptogenesis*, *JCAP* **04** (2006) 004 [[hep-ph/0601083](#)];  
 A. Abada et al., *Flavour matters in leptogenesis*, *JHEP* **09** (2006) 010 [[hep-ph/0605281](#)];  
 E. Nardi, Y. Nir, E. Roulet and J. Racker, *The importance of flavor in leptogenesis*, *JHEP* **01** (2006) 164 [[hep-ph/0601084](#)];  
 see also, R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, *Baryogenesis through leptogenesis*, *Nucl. Phys. B* **575** (2000) 61 [[hep-ph/9911315](#)];  
 T. Endoh, T. Morozumi and Z.-h. Xiong, *Primordial lepton family asymmetries in seesaw model*, *Prog. Theor. Phys.* **111** (2004) 123 [[hep-ph/0308276](#)];  
 T. Fujihara et al., *Cosmological family asymmetry and CP-violation*, *Phys. Rev. D* **72** (2005) 016006 [[hep-ph/0505076](#)];  
 S. Antusch, S.F. King and A. Riotto, *Flavour-dependent leptogenesis with sequential dominance*, *JCAP* **11** (2006) 011 [[hep-ph/0609038](#)];  
 O. Vives, *Flavoured leptogenesis: a successful thermal leptogenesis with  $N_1$  mass below  $10^8$  Gev*, *Phys. Rev. D* **73** (2006) 073006 [[hep-ph/0512160](#)];  
 S. Pascoli, S.T. Petcov and A. Riotto, *Connecting low energy leptonic CP-violation to leptogenesis*, [hep-ph/0609125](#);  
 G.C. Branco, R. Gonzalez Felipe and F.R. Joaquim, *A new bridge between leptonic CP-violation and leptogenesis*, [hep-ph/0609297](#).
- [25] K. Turzyski, *Degenerate minimal see-saw and leptogenesis*, *Phys. Lett. B* **589** (2004) 135 [[hep-ph/0401219](#)];  
 Y.H. Ahn, C.S. Kim, S.K. Kang and J. Lee,  $\mu - \tau$  symmetry and radiatively generated leptogenesis, [hep-ph/0610007](#);  
 R. Gonzalez Felipe, F.R. Joaquim and B.M. Nobre, *Radiatively induced leptogenesis in a minimal seesaw model*, *Phys. Rev. D* **70** (2004) 085009 [[hep-ph/0311029](#)];  
 G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and B.M. Nobre, *Enlarging the window for radiative leptogenesis*, *Phys. Lett. B* **633** (2006) 336 [[hep-ph/0507092](#)].
- [26] V.D. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, *Bi-maximal mixing of three neutrinos*, *Phys. Lett. B* **437** (1998) 107 [[hep-ph/9806387](#)].
- [27] M. Raidal, *Prediction  $\Theta_C + \Theta_{\text{sol}} = \pi/4$  from flavor physics: a new evidence for grand unification?*, *Phys. Rev. Lett.* **93** (2004) 161801 [[hep-ph/0404046](#)];  
 H. Minakata and A.Y. Smirnov, *Neutrino mixing and quark lepton complementarity*, *Phys. Rev. D* **70** (2004) 073009 [[hep-ph/0405088](#)];

- J. Ferrandis and S. Pakvasa, *QLC relation and neutrino mass hierarchy*, *Phys. Rev. D* **71** (2005) 033004 [[hep-ph/0412038](#)];
- S.K. Kang, C.S. Kim and J. Lee, *Quark-lepton complementarity with renormalization effects through threshold corrections*, *Phys. Lett. B* **619** (2005) 129 [[hep-ph/0501029](#)];
- N. Li and B.-Q. Ma, *Unified parametrization of quark and lepton mixing matrices*, *Phys. Rev. D* **71** (2005) 097301 [[hep-ph/0501226](#)];
- K. Cheung, S.K. Kang, C.S. Kim and J. Lee, *Lepton flavor violation as a probe of quark-lepton unification*, *Phys. Rev. D* **72** (2005) 036003 [[hep-ph/0503122](#)];
- Z.-z. Xing, *Nontrivial correlation between the CKM and MNS matrices*, *Phys. Lett. B* **618** (2005) 141 [[hep-ph/0503200](#)];
- A. Datta, L. Everett and P. Ramond, *Cabibbo haze in lepton mixing*, *Phys. Lett. B* **620** (2005) 42 [[hep-ph/0503222](#)].
- [28] S. Antusch and S.F. King, *Charged lepton corrections to neutrino mixing angles and CP phases revisited*, *Phys. Lett. B* **631** (2005) 42 [[hep-ph/0508044](#)].